III. *An Extract of a Letter from the Reverend Dr.* John Wallis *to* Richard Waller, *Esq; Secretary to the Royal Society, concerning the Spaces in the* Cycloid, *which are perfectly Quadrable.*

Oxford, August 22. 1695.

SIR,

I Find it is thought by moſt, that there is no other part of the *Semicycloid* Figure (adjacent to the Curve) that is capable of being Geometrically Squared, but theſe two, *viz.*

1. The Segment AbV, Figur. 1. taking $AV = \frac{1}{4} A\alpha$, (which was firſt obſerved by Sir *Chriſtopher Wren*, and after him by *Hugenius* and others) and it is $= \frac{3}{2} s R = \frac{3}{4} R^2 \sqrt{3}$.

2. The Trilinear AdD (taking dD, in the Parallel dDC, paſſing through the Center C,) which is $= R^2$.

But it is otherwiſe (as I have ſhewed in my Treatiſe *De Cycloide*, and that *De Motu*; the Figures of which latter I retain here, ſo far as they concern this occaſion;) there being other portions of it, equally capable of Quadrature.

In order to which, I there ſhew (*De Motu, Cap.* 5. *Prop.* 20. *A. pag.* 802, 803, 804.) that not only the *Cycloid* is Triple to the Circle Generant, (which was known before) but that the *reſpective parts* of that are Triple to thoſe of this: which is the Foundation on which I build my whole Proceſs concerning the *Cycloid* in both Treatiſes, (and which is not pretended, that I know of, to have been obſerved, or known, by any body before me:) That is $b\beta\alpha A$ *Figur.* 1, triple to the Sector $B\alpha A$ (taking $b\beta$ parallel to $B\alpha$) where ever, in the Curve $A\tau$, we take the point b.

I then

I then ſhew, that the *Cycloid* is a Figure compounded of theſe two; the Semicircle *ADα*, and the Trilinear *ADατbA*, lying between the two Curves *ADα* and *Adτ*, (and therefore, to Square any part of theſe, is the ſame as to Square the reſpective part of the *Cycloid.*

I ſhew further (*Ibidem, pag.* 804.) that this Trilinear is but a diſtorted Figure, (by reaſon of the Semicircle thruſt in between it and its Axis) which being reſtored to its due Poſition (by taking out the Semicircle into a different Figure, as *Figur.* 2. and thruſting the Lines *bB* home to the Axis, ſo as that *BV* be the ſame point) is the ſame with *Aτα*, *Figur.* 3. (the Parallelograms *bβαB* being ſet upright, which in the *Cycloid* ſtand ſloping; and the Circular Archs *bβ*, *Figur.* 1, becoming ſtreight-lines in *Figur.* 3. and the Lines *bB* being, in both, equal to the reſpective Archs *BA*, every where;) which therefore I call *Trilineum Reſtitutum* (the Trilinear reſtored to its due Poſition, which Figure I do not find that any before me has conſidered:) So that to Square any part of this, is the ſame as to Square the reſpective part of the *Cycloid*, (or of the Trilinear in the *Cycloid*:) That which in the *Cycloid* lies between two Archs of the Circle Generant in different Poſitions, anſwering to that which, in the reſtored Figure, lies between the reſpective ſtreight-lines.

And therefore $AdDA, = \tau d\delta\tau$, *Figur.* 1, $= AdDA = \tau d\delta\tau$, *Figur.* 3, $= R^2$. And $AbkdA$, $\tau bk\delta\tau$, *Figur.* 1, $= AbkdA$, $\tau bk\delta\tau$, *Figur.* 3, $= sR$. And bkd, *Figur.* 1, $= bkd$, *Figur.* 3, $= R^2 - sR$, *Ibidem, Cap.* 17. *B. pag.* 756. Where, if *b* be taken above *dkDC* (paſſing through the Center *C*,) theſe Figures are within the *Cycloid*, and within the reſtored Figure; but without them, if *b* be taken below that Line, and adjacent to the Curve *Abτ*, in both caſes.

By *R*, I underſtand the Radius of the Circle Generant; and by *s*, the Right Sine of the Arch *B A*, whoſe Verſed Sine is *V A*.

And, where ever in my whole Diſcourſe of the *Cycloid*, or the Reſtored Trilinear (which is a Figure of Archs, and a Figure of Verſed Sines) the Arch *a* is no Ingredient in the deſignation; ſuch part or portion of them is capable of being Geometrically ſquared. But when I exclude *a*, I do therein exclude *P* (for that is an Arch alſo) and $f = a + s$, and $e = a - s$, becauſe *a* is therein included.

Mr. *Caſwell*, (not being aware that I had ſquared theſe Figures) had done the ſame by a Method of his own, (which he ſhewed me lately) which I would have inſerted here, but that he thought it not neceſſary; and inſtead thereof, hath given me the Quadrature of a Portion of the *Epicycloid* (which you will receive with this) and, I think, it is purely new.

IV. *The Quadrature of a Portion of the Epicycloid. By Mr.* Caſwel.

SUppoſe *D P V* to be half of an exterior Epicycloid, *V B* its Axis, *V* the Vertex, *V L B* half of the generant Circle, *E* its Center; *D B* the Baſe, *C* its Center: Biſect the Arc of the Semicircle *V B* in *L*, and on the Center *C* through *L* draw a Circle cutting the Epicycloid in *P*: Then I ſay the Curvilinear Triangle *V L P* will be $= BEq$ in $\frac{CE}{CB}$; that is, the Square of the Semidiameter of the generant Circle will be to the Curvilinear Triangle *V L P*, as *C B* the Semidiameter of the Baſe, to *C E*: which *C E* in the exterior Epicycloid is the

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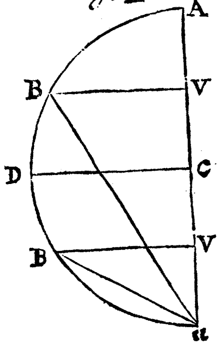
Fig. 2



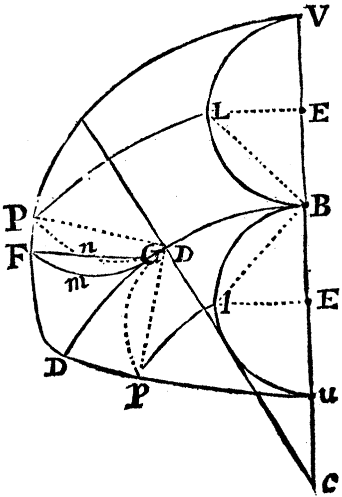


Fig. I

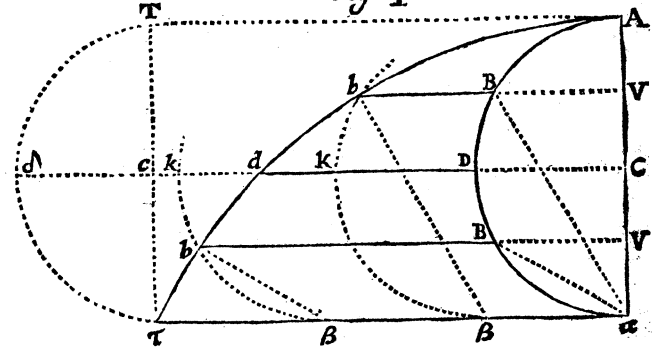


Fig. 3